

A high frequency cascade thermoacoustic engine

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Abstract

A miniature cascade thermoacoustic engine, which consisted of one standing-wave stage and one traveling-wave stage in series, was built and tested, which length was about 1.2 m, operating at 470 Hz using helium as working gas. The cascade modeling, the simulation and the primary experimental results are described in this paper. Four different configurations of the miniature cascade thermoacoustic engines had been designed and compared. According to the analysis, the diameter ratio of stages was designed to extend the traveling-wave region, which optimized value was about 1.69. The peak-to-peak value of the acoustic pressure was predicted to arrive to 3 bar at the 3 MPa mean pressure of helium when 300 W heating power was the input. The features of the engine were predicted delivering 68 W acoustic power with a thermal efficiency of up to 22.74% (the ratio of acoustic power to heater power). Due to careful designing, the engine self-excited the oscillation smoothly from the first experiment. An onset temperature gradient of about 4.5 K/mm was achieved, and the peak-to-peak acoustic pressure was 48 KPa at the 2 MPa mean pressure when 200 W heating power was the input. The design computation and experimental results showed a rather good agreement between the measured and calculated pressure phasor and temperatures distributions in the cascade thermoacoustic engine.

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1. Introduction

In recent years, a new type of hybrid thermoacoustic devices are being developed in order to improve the efficiencies to the levels that can rival what can be obtained from internal combustion engine [1]. The elemental thermoacoustic devices are divided into standing-wave type and traveling-wave type. The former depends on the phase lagging between heat transfer and the pressure oscillation in order to accomplish a similar Brayton cycle, which was achieved by the irreversible heat transfer between the working fluid and the stack. The phasing between pressure and velocity within the latter regenerator was the same as the phasing in a Stirling's regenerator, so the latter operates as the Stirling cycle with perfect heat transfer. Because

there is obvious irreversible entropy production in the former, the efficiency of the former is usually limited to only about 2/3 that of the latter [2,3]. Due to the low acoustic impedance of working gas in a pure traveling-wave region, the actual efficiency of pure traveling-wave engine was low for the serious viscosities and heat dissipation. Until 1999, a thermoacoustic-Stirling hybrid engine, a looped tube joined with a branch resonator, developed by Swift etc., yielded 42% of the Carnot efficiency. This toroidal topology was not easy to fabricate, and in addition, it needed to provide an adjustable jet pump to suppress Gedeon streaming around the torus. To eliminate Gedeon streaming, and make the fabrication economical as well as to increase efficiency simultaneously, a cascaded thermoacoustic engine was built in 2003 and was experimented in the latter, which operated at 25 Hz with a working gas of argon [4,5]. The cascade thermoacoustic device was formed with plurality of thermoacoustic units which were cascaded together within the region of high specific

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acoustic impedance in a resonator system, where at least one of the thermoacoustic units was a regenerator unit. The practical acoustic field in the cascade thermoacoustic engine was a commixture of standing-wave and traveling-wave in such a half-wave resonance system. There existed a traveling-wave region that had high specific impedance. The regenerator unit could be arranged in this traveling-wave region, which would upgrade the efficiency of the whole system. Due to the cascaded thermoacoustic engine which had more uncertain boundary condition as well it having more than one stage, the design of such an engine was much more difficult to configure and adjust. On the other hand, miniature thermoacoustic engines are being developed in order to increase the energy density and compactness simultaneously. A miniature cascade thermoacoustic device predicted more applications, such as thermal engine, refrigerator, heat pump and so on. It is well known that as the size of a thermoacoustic device decreases, the working frequency will increase, and there are more difficulties in designing such a miniature cascade device. After many efforts on designing, a miniature two-stage cascade thermoacoustic engine was chosen as our first model. The manufactured cascade thermoacoustic engine was 1.2 m long, operating at 470 Hz with helium as the working gas. The exploration of such a miniature cascade device is presented in this paper. Design computation and experimental results showed a rather good agreement between the measured and calculated pressure phasor and temperatures distributions in the engine.

2. Cascade modeling

The cascade thermoacoustic engine worked in half-wavelength resonant mode. The basic cascade thermoacoustic engine was configured with both-end-cavities, shown in Fig. 1, and was composed of one standing-wave component and one traveling-wave component. In the acoustic field of a cascade engine, the phase, pressure leading velocity, changed from negative 90° to positive 90° near the pressure antinode, where the acoustic impedance was very high, which was satisfied the two requirements for efficient thermoacoustic transformation, the traveling-wave phase and the high impedance. So the regenerator-based unit

could well amplify acoustic power in the so called traveling-wave region, and that acoustic feedback was received by a prepositive stack-based unit.

To investigate the half-wavelength resonance mode for cascade thermoacoustic units, thermoacoustic approximation to the momentum equation, the continuity equation and the enthalpy streaming equation were adopted. According to the linear thermoacoustic theory, these were the following essential equations [6]:

$$\begin{cases} dp_1 = -\frac{i\omega\rho_m dx/A}{1-f_v} U_1 \\ dU_1 = -\frac{i\omega A dx}{\gamma p_m} [1 + (\gamma-1)f_k] p_1 + \frac{(f_k - f_v)}{(1-f_v)(1-\sigma)} \frac{dT_m}{T_m} U_1 \\ \frac{dE_2}{dx} = \frac{1}{2} \text{Re} \left[\tilde{U}_1 \frac{dp_1}{dx} + \tilde{p}_1 \frac{dU_1}{dx} \right] \\ \langle T_1 \rangle = \frac{1}{\rho_m c_p} (1-f_k) p_1 - \frac{1}{i\omega A} \frac{dT_m}{dx} \frac{(1-f_k) - \sigma(1-f_v)}{(1-f_v)(1-\sigma)} U_1 \end{cases} \quad (1)$$

where p_1 is acoustic pressure amplitude, U_1 is the volume oscillating velocity amplitude, ω is the angle frequency, ρ_m is the density, p_m is the mean pressure, A is the cross-sectional area of the channel for the working fluid, γ is the ratio of specific heats, f_v , f_k are the viscous and thermal-relaxation spatial average complex geometry factors, E_2 is the acoustic power and T_1 is the oscillating temperature. The resonator and thermoacoustic units (including stack, regenerator, heat exchanger and thermal buffer tube) can be modelled by the above equations.

The acoustic power produced in the standing-wave unit was estimated approximately as follows:

$$\begin{aligned} E_{2s} &= \frac{1}{2} |p_1|^2 \omega A \Delta x \frac{\gamma-1}{\gamma p_m} \text{Im}(-f_k) \left(\frac{(T_H - T_C)/\Delta x}{\nabla T_{\text{crit}}} - 1 \right) \\ \nabla T_{\text{crit}} &= \frac{|p_1|/\rho_m c_p}{|U_1|/A\omega} \end{aligned} \quad (2)$$

where T_H and T_C are the hot and the ambient temperatures of the fluid in the stack, respectively. If the temperature gradient was steep enough, the acoustic power was produced.

The amplified acoustic power by the second traveling-wave regenerator is shown by the following formula:

$$\Delta E_2 = E_{2s} \left(\frac{T_H}{T_C} - 1 \right) - \frac{3\mu\Delta x}{2Ar_H^2} |U_1|^2 - \frac{\omega^2 Ar_H^2}{6kT_m} |p_1|^2 \quad (3)$$

The acoustic power gain of a traveling-wave stage is approximately proportional to the ratio of hot temperature to ambient temperatures.

The efficiency was often about 20% for the standing-wave engine and 33% for the traveling-wave engine, the total efficiency for the cascade engine, composed of one standing-wave stage and one or more traveling-wave stages, was the function of stage number and the ratio of the hot temperature to the ambient temperatures [4]. The

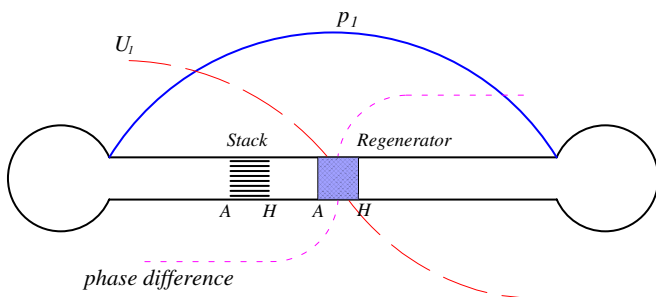


Fig. 1. The half-wavelength resonance system with both-end-cavities.

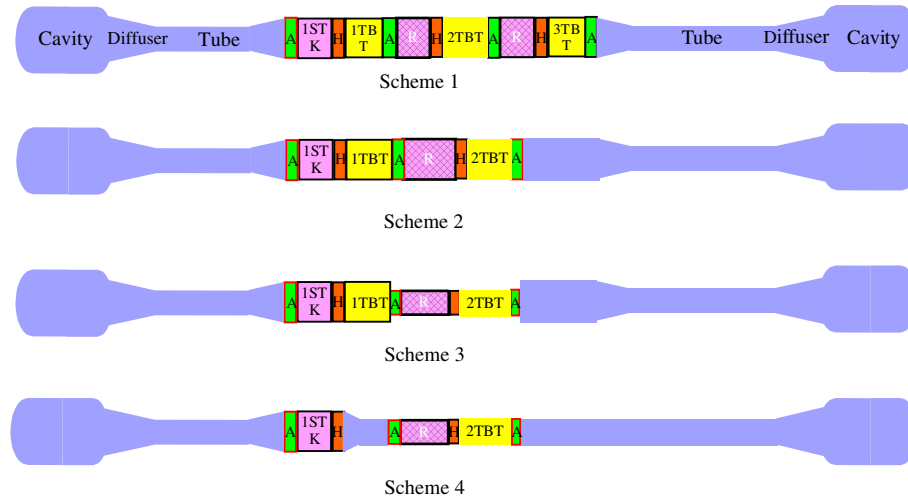


Fig. 2. Different configurations of the high frequency cascade thermoacoustic engine.

number of stages determines the total efficiency of the cascade engines. Three or four stages were suggested by Swift, but it was often limited to two or three stages in a high frequency system, because the wave length was shorter and the real traveling-wave region was even shorter in such half-wavelength region, which was only a few percent of the resonator length in ordinary systems. For a practical cascade thermoacoustic engine, for example, when the ratio of the hot temperature to the ambient temperatures was about 2.5 in ordinary condition, the theoretical cascade efficiency was 26% for two stages and 30% for three stages.

According to the modeling, the numerical simulative model of a cascade thermoacoustic engine was built by DeltaE [7]. Due to the complicated boundary conditions and coupling effects between stages, the simulation and tuning were complicated. The locations and dimensions of different thermoacoustic units must be optimized to achieve more acoustic power on a higher efficiency level. Due to the results of the reciprocity of stages, the cascade performance was very sensitive to the location and the length of the traveling-wave region, so the length of the traveling-wave should be designed to as long as possible, which is discussed in the next section in detail.

3. Configuration and comparison

According to the analysis of the cascade modeling, only the resonator with both-end-cavities could realize the cascade arrangement, which nearly corresponded to an effective half-wavelength tube. The basic geometry arrangement is as shown in Fig. 1. There were different stage numbers to arrange as well as different matching between stages in the same basic resonator. Because the length of the traveling-wave region was only a few percent of the total size, the configuration consisting of two or three stages was considered. To prolong the traveling-wave region, a method of tuning the diameter ratio of stages was adopted, which was feasible and effective. From the lumped impedance network

approximately, the inertance near the pressure antinode could be ignored, because the region was close to the velocity node. The local acoustic compliance, being proportional to the local diameter, mostly determines the phasing property in the region. From Fig. 3 we can deduce the length of the traveling-wave region which would be greater when the compliance was smaller, which is testified in the numerical simulation. However, with smaller diameters, the dissipation of the regenerator would increase and the heating power would decrease. So the diameter ratio of the stages was pivotal, which was about 1.69 from the optimized simulation.

Four different detailed configurations, shown in Fig. 2, were designed and compared, where A denoted the ambient heat exchanger, H denoted the hot heat exchanger, STK denoted the stack of the standing-wave stage, R denoted the regenerator, TBT denoted the thermal buffer tube, and the number denoted the order of the stages. The performance for every scheme is shown in Tables 1

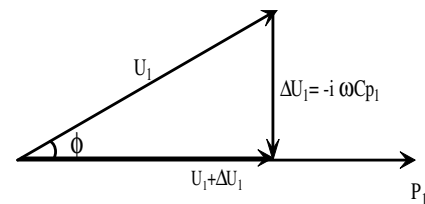


Fig. 3. The influence of the local compliance on traveling-wave phasor.

Table 1
The efficiency and acoustic power for different configurations

	$\eta_1\%$	$\eta_{1c}\%$	$\eta_2\%$	$\eta_{2c}\%$	$\eta_3\%$	$\eta_{3c}\%$	$\eta_t\%$	E_{2W}
No.1	17.57	31.70	10.22	29.40	11.01	22.03	11.75	68.55
No.2	14.06	24.24	20.00	31.06	—	—	12.14	36.41
No.3	16.13	27.51	28.92	45.51	—	—	18.50	55.49
No.4	19.98	31.93	34.06	53.43	—	—	22.74	68.22

Table 2

The heat power and temperature at the hot end of the regenerators for different configurations

	Q_{1stW}	$T_{H1}K$	Q_{2ndW}	$T_{H2}K$	Q_{3rdW}	$T_{H3}K$
No.1	300	820	100	460	186	600
No.2	200	656	100	843	–	–
No.3	200	700	100	823	–	–
No.4	200	801.75	100	827.65	–	–

and 2, where T_H is the temperature at hot end of stages, η is efficiency, η_C is the Carnot efficiency, η_t is the total efficiency and Q is heating power. The two tables describe the efficiencies and acoustic powers to export for different configurations at nearly the same heating power and temperature gradient. The first scheme constituted of one standing-wave unit and two regenerator-based units, which were located at the symmetrical position from the sweet spot. Because the length of the traveling-wave region was short, the total efficiency was not high, which meant that more acoustic power was dissipated in the regenerators. The latter three schemes were all composed of one standing-wave unit and one regenerator-based unit. The efficiency of the third scheme was higher than that of the second due to its longer traveling-wave region. The fourth scheme was perfect because of some gradual and smooth diameter change avoiding flow separation, which was chosen to fabricate the prototype of the high frequency cascade thermoacoustic engine.

Fig. 4 shows the simulated acoustic pressure and velocity of the fourth scheme for a typical operating point. The amplitude of pressure was at 5% near the pressure anti-

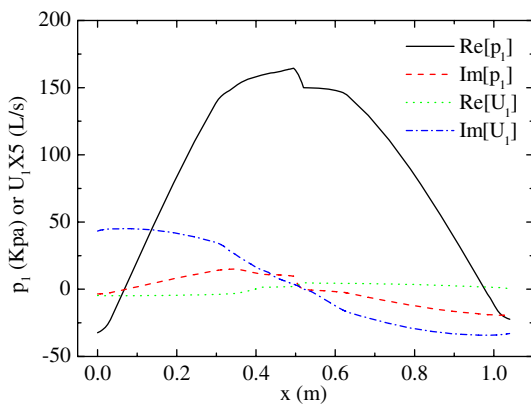


Fig. 4. The simulated components of pressure and volume velocity for a typical operating point using DeltaE.

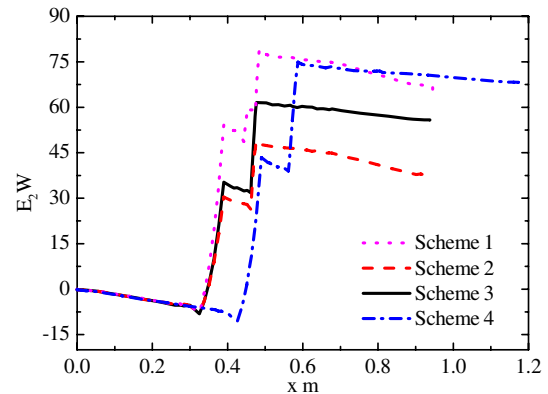


Fig. 5. The acoustic power distribution for different schemes.

node. The wave qualitatively resembled a half-wavelength standing-wave, but the imaginary part of pressure was not zero. The U_1 phasors changed from positive 90° to negative 90° along the axial. But the p_1 phasors in the thermoacoustic units region changed by less than 10° . So the phasor of the pressure leading velocity was near the traveling-wave phasor around the sweet spot, which was just the motivation to form the cascade idea.

Fig. 5 shows the acoustic power distribution for different schemes. The first sharp step change indicated that power was produced in the stack, and the other step change indicated that power was then amplified in the regenerator. The power was negative above the first stage, that displayed the dissipation of acoustic power in the delivery progress to the upper cavity. The dot line denoted the three stages (scheme 1) and the dash dot line denoted the two stages (scheme 4) with optimized traveling-wave region and smooth transition in diameters. It was clear that the acoustic power exported from the fourth scheme was almost equal to that of the first scheme, at a higher efficiency of up to 22.74%, so the fourth scheme was adopted in fabrication.

4. Experimental apparatus

The prototype of the miniature cascade thermoacoustic engine, shown in Fig. 6, had two thermoacoustic engine stages in the middle part of the half-wave resonator with only one PAN (pressure antinode) at the navel. The diameter varied from segment to segment along the apparatus so as to minimize acoustic dissipation in the resonator, which was based on the design simulation. The 100 mm long compliance cavities were set on both the ends symmetrically, connected with the resonator tube by tapered tube diffusers

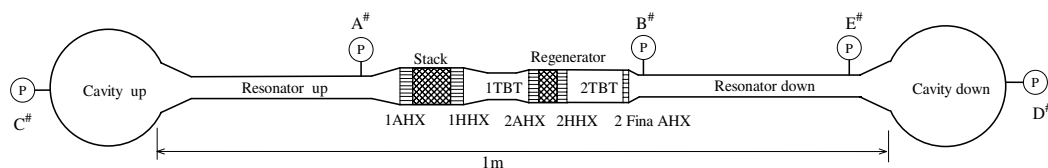


Fig. 6. Schematic of the miniature cascade thermoacoustic engine.

to minimize dissipation of acoustic power. In the standing-wave stage, the stack, composed of 60 mesh stainless steel screens, was sandwiched with the ambient heat exchanger (AHX) and hot heat exchanger (HHX). The AHX was a plate type heat exchanger, with helium gas oscillating in the spaces of the plates, and ambient temperature water flowing in the round jacket spaces surrounding the fins. The HHXs consisted of ceramic honeycombs through which the long electric heating filament was wound. In the similar second stages, traveling-wave stage, the regenerator was filled with 200 or 300 mesh steel stainless screens. The first thermal buffer tube (TBT) connected the two stages smoothly. And the second TBT connected the second stage and the final ambient heat exchanger. To insulate the AHX from adjacent HHX, the length of these TBTs must be of the order of 10 times the gas displacement amplitude [4]. The dimensions of different AHXs, such as length and fins spacing, were determined by its heat power, local oscillation velocity and thermal penetration depth of the working gas, individually.

The two stages were enclosed within the vacuum insulation envelope (not shown in Fig. 6) except the ambient heat exchangers. The locations of the five pressure transducers labeled with capital letters are denoted in Fig. 6, with the three in the middle distributed in equal interval. The six thermocouples (not shown in Fig. 6) are located on the bottom, the center and the top of the stack and regenerator, respectively, and the other two thermocouples are set on both extreme cavities. The heating powers were supplied by DC power source, and measured by the read values of volts and currents of the DC source.

5. Experimental results and discussion

The prototype of the cascade thermoacoustic engine had been tested using helium as the working gas at different mean pressures. The aim was to observe the basic performance of the cascade engine such as the onset conditions, frequency and the amplitude of pressure. The heating powers of the two stages were adjusted by hand until the oscillation was produced and then developed to a steady state. In this progress, the temperatures on the hot end of the stages were controlled below 550 °C to ensure the safety of the wall material of the cascade thermoacoustic engine. To study the features of the cascade thermoacoustic engine, the mean pressure (noted as p_m) is set into ten levels from 1.2 MPa to 2.6 MPa, to investigate the effects of the onset temperature, resonator frequency and acoustic amplitude and so on.

The cascade efficiency increased with the heating power of the second engine stage, which meant more contribution for the efficiency from the regenerator. So the heating power of the second engine stage (noted as Q_2) should be input as much as possible, but the heating power of the first engine stage (noted as Q_1) should also be kept on a certain level. The amplified acoustic power in the regenerator is shown as the formula (3) [6]. It was clear that the delivered

acoustic power was proportional to the power E_{2S} produced at the standing-wave stage. To compromise between the high efficiency and great acoustic power, Q_1 should achieve appropriate levels. According to some beforehand experimental results, Q_1 was set at about 100 W and Q_2 was set at about 140 W.

By designing well, the engine was easy to start with self-excited oscillation in our experiments, which temperature grad was about 4.5 K/mm, although it was often a difficult case for this type of engine [4]. Fig. 7 shows the stable pressure waveforms at the four different locations operating under a typical condition. The difference of phasor and amplitude of pressure between A and B was very small, which was 10° in ordinary. The pressure phasor of the center was almost contrary to that of both the end-cavities. Fig. 8 was the FFT analysis of the pressure shown in Fig. 7. The disagreements between measured and calculated frequency was about 0.5%.

The agreement between the measured and calculated pressure phasors is shown in Fig. 9. The agreement between them would yield confidence in the simulated volume–velocity phasors. It displayed also a small difference in time phasor of pressure at the two stages, from which the PAN location could be predicted. Because the cross area of the tube varied abruptly at different locations

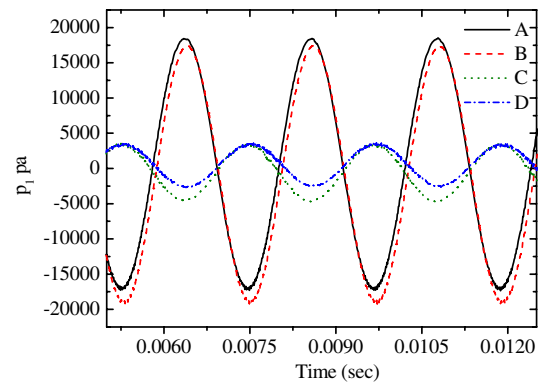


Fig. 7. The pressure waveform at different locations for a typical operating point.

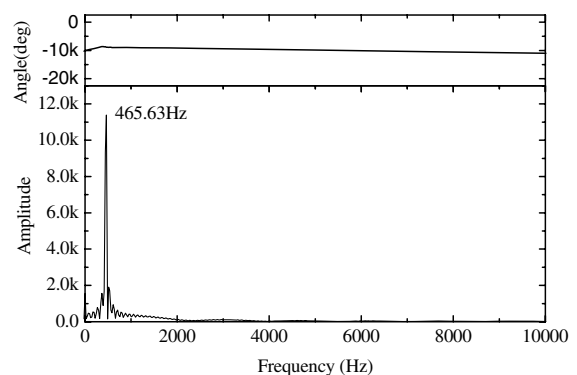


Fig. 8. The FFT analysis of the pressure for the same operating point as Fig. 7.

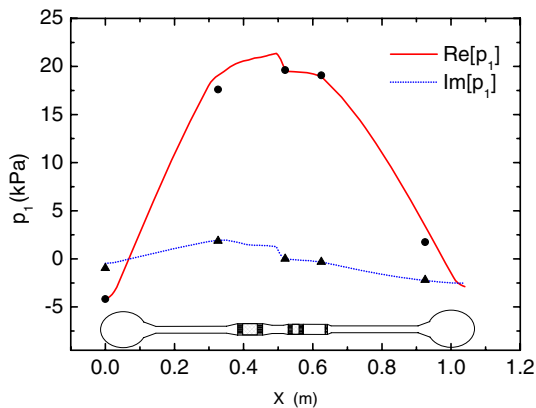


Fig. 9. Components of pressure for a operating point. Points were measured values, and lines were calculations by DeltaE. The center points phase is taken to be zero and is not measured.

or the boundary compliances were inadequate, the wave occurred at a little aberration from the simple harmonic wave. This aberration of the waves increased the difficulty of the onset progress, which could be concluded from our experiments as well as from Swift's experiments.

Nearly linear temperature profile in the stack and regenerator, shown as Fig. 10, were measured and accorded with the design prediction. It could be predicted that there was no evident acoustic streaming from the temperature profile.

The mean pressure p_m had the most significant effect on the amplitude of pressure and onset temperature. Figs. 11 and 12 show the influencing trend of p_m , compared under a certain operating mode. The lower onset temperature and higher amplitude of pressure were obtained on the higher mean pressure level.

Another series of experiments on the single stage standing-wave engine were also made in order to testify the contribution of the traveling-wave engine. The two-stage amplitude of pressure was enhanced to about 30% to 50% than that produced by a single standing-wave stage. And the cascade engine had lower onset temperature compared to the single standing-wave stage, and even decreased

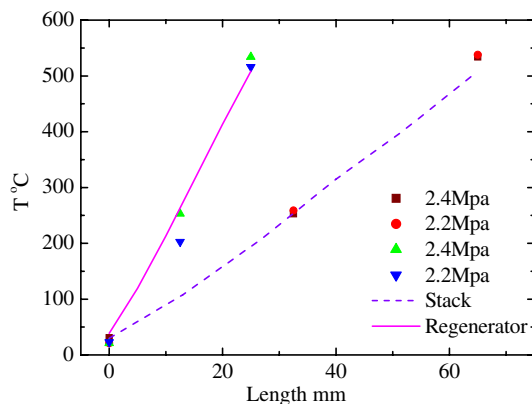


Fig. 10. The linear temperature profile in the stack and regenerator. Points were measured and lines are calculated.

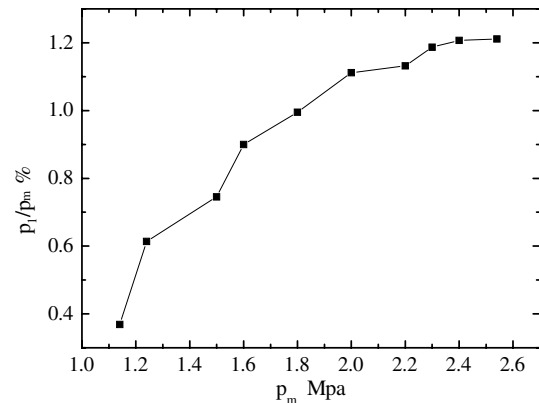


Fig. 11. The mean pressure was how to affect the amplitude of pressure.

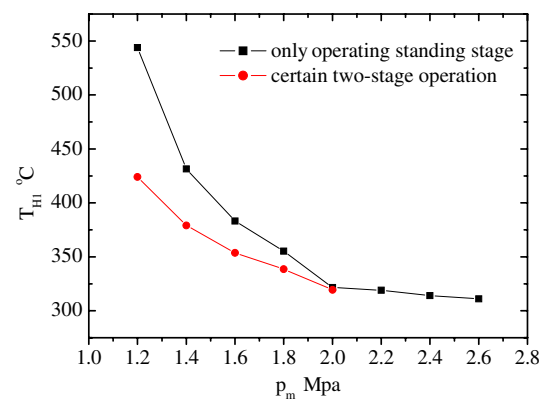


Fig. 12. The mean pressure was how to affect the onset temperature.

to 100 °C in some instances, because the temperature gradient in the second stage would overcome some loss in the regenerator before onset.

The most disagreement between design and experiment was the amplitude of pressure, which was 5% and 1.25%, respectively. The acoustic power produced in the first standing-wave stage was less than that calculated according to the formula (2) [6].

This might be due to that Q_1 was overestimated more than 50% in the design, and also due to the unfitted structure of the first HHX. So the acoustic power would decrease about 50% even with the same efficiency. Furthermore, the experiments using nitrogen as working gas in the cascade engine had obtained 3.3% amplitude for the same operation conditions, operating at 165 Hz, with less thermal penetration depth. In further experiments, the stack and HHX will be modified.

6. Conclusions

A prototype of the high frequency cascade thermoacoustic engine was built and tested, which simple topology eliminated Gedeon streaming and enhanced efficiency. According to the simulation analysis and experiments, it could be concluded as following:

1. The important issue of the designing of the cascade engine was to configurate a proper resonance system with at least one region of high acoustic impedance and near traveling-wave phasor.
2. The performance of cascade engine was very sensitive to the length of the traveling-wave region. The total efficiency was increased with the cascade number, which was limited by the length of the traveling-wave region. Tuning the diameter ratio of the stages was an effective method to prolong the length of the traveling-wave region, which optimum value was about 1.69.
3. For eliminating the aberration of the waves, in our design, it was achieved by adjusting the size of the stages. So the onset of the self-excited oscillation was easy to accomplish in our prototype of the cascade thermoacoustic engine.
4. The design and experiments showed a rather good agreement, such as frequency, pressure phasor and temperatures distributions. The only disagreement in the amplitude of pressure was due to some unfitted constructions of HHX and the stack.
5. The mean pressure p_m had the most significant effect on the amplitude of pressure and onset temperature gradient.

Acknowledgements

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